

Using `asremlPlus`, in conjunction with `asreml`, to do the analysis of a wheat experiment that includes choosing a local spatial variation model using AICs

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This vignette shows how to use `asremlPlus` (Brien, 2023), in conjunction with `asreml` (Butler et al., 2020), to select the terms to be included in a mixed model for an experiment that involves spatial variation by comparing of information criteria . It also illustrates diagnostic checking and prediction production and presentation for this experiment. Here, `asremlPlus` and `asreml` are packages for the R Statistical Computing environment (R Core Team, 2023).

It is divided into the following main sections:

1. Set up the initial model for this experiment
2. Compare a series of information criteria to select a linear mixed model for the data
3. Diagnostic checking using residual plots and variofaces
4. Prediction production and presentation

1. Set up the initial model for this experiment

```
library(asreml, quietly=TRUE)

## Online License checked out Mon Jun 12 17:43:51 2023

library(asremlPlus)
suppressMessages(library(qqplotr, quietly=TRUE))
options(width = 100)
```

Get data available in `asremlPlus`

The data are from a 1976 spring wheat experiment and are taken from Gilmour et al. (1995). An analysis is presented in the `asreml` manual by Butler et al. (2020, Section 7.6), although they suggest that it is a barley experiment.

```
data(Wheat.dat)
```

Add row and column covariates for the spatial modelling

```
tmp.dat <- within(Wheat.dat,
  {
    cColumn <- dae::as.numfac(Column)
    cColumn <- cColumn - mean(unique(cColumn))
    cRow <- dae::as.numfac(Row)
```

```
cRow <- cRow - mean(unique(cRow))
})
```

Fit an initial model - Row and column random

In the following, an initial model is fitted that has the terms that would be included for a balanced lattice. In addition, a term WithinColPairs has been included to allow for extraneous variation arising between pairs of adjacent lanes.

```
current.asr <- do.call(asreml,
  list(yield ~ Rep + WithinColPairs + Variety,
    random = ~ Row + Column,
    residual = ~ Row:Column,
    data = tmp.dat))
```

```
## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:43:51 2023
##           LogLik      Sigma2      DF      wall      cpu
##  1      -702.701      26232.20     119 17:43:51     0.0
##  2      -700.484      24394.12     119 17:43:51     0.0
##  3      -698.491      22487.75     119 17:43:51     0.0
##  4      -697.584      21242.11     119 17:43:51     0.0
##  5      -697.367      20590.05     119 17:43:51     0.0
##  6      -697.359      20477.28     119 17:43:51     0.0
```

Intialize a model sequence by loading the current fit into an asrtests object

In creating the asrtests object, IClikelihood is set to full so that the full Restricted Maximum Likelihood (full REML) of Verbyla, 2019 is incorporated into the tests.summary of the asrtests object.

```
current.asrt <- as.asrtests(current.asr, NULL, NULL, IClikelihood = "full",
  label = "Initial model")
```

```
## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged
## Calculating denominator DF
```

Check for and remove any boundary terms and print a summary of the fit in the asrtests object

```
current.asrt <- rmboundary(current.asrt)
print(current.asrt)
```

```
##
##
## ##### Summary of the fitted variance parameters
##
##           component std.error  z.ratio bound %ch
## Row           5943.898  3815.514  1.557824    P 0.0
## Column        12380.527  6323.542  1.957847    P 0.3
## Row:Column!R  20477.280  2896.642  7.069316    P 0.0
##
##
## ##### Pseudo-anova table for fixed terms
```

```
##
##
## Wald tests for fixed effects.
## Response: yield
##
##           Df denDF   F.inc    Pr
## (Intercept)  1  14.9 1390.00 0.0000
## Rep          5  25.3   6.04 0.0008
## WithinColPairs 1  10.4   0.49 0.4998
## Variety      24 104.8   4.71 0.0000
##
##
## ##### Sequence of model investigations
##
## (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
##
##           terms DF denDF  p      AIC      BIC      action
## 1 Initial model 31      3 NA 1720.891 1823.253 Starting model
```

The `test.summary` output shows that no changes have been made to the model loaded using `as.asrtests`. The pseudo-anova table shows that Varieties are highly significant ($p < 0.001$)

2. Compare a series of information criteria to select a linear mixed model for the data

In this section, models are compared using Akaike Information Criterion (AICs) based on the full REML.

Check the need for the term for within Column pairs (a post hoc factor)

```
current.asrt <- changeModelOnIC(current.asrt, dropFixed = "WithinColPairs",
                                label = "Try dropping withinColPairs", ICLikelihood = "full")

## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:43:52 2023
##           LogLik      Sigma2      DF      wall      cpu
## 1          -697.359    20472.09    119 17:43:52    0.0

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged

## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:43:52 2023
##           LogLik      Sigma2      DF      wall      cpu
## 1          -697.359    20472.09    119 17:43:52    0.0
## 2          -697.359    20472.08    119 17:43:52    0.0
## Calculating denominator DF
## Calculating denominator DF

print(current.asrt)

##
##
## ##### Summary of the fitted variance parameters
##
##           component std.error  z.ratio bound %ch
```

```
## Row          5941.153  3813.586 1.557891    P    0
## Column       11165.335  5583.267 1.999785    P    0
## Row:Column!R 20472.402  2895.582 7.070219    P    0
##
##
## ##### Pseudo-anova table for fixed terms
##
##
## Wald tests for fixed effects.
## Response: yield
##
##           Df denDF    F.inc    Pr
## (Intercept) 1  15.3 1466.00 0e+00
## Rep          5  26.7   6.11 7e-04
## Variety      24 105.3   4.73 0e+00
##
##
## ##### Sequence of model investigations
##
## (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
##
##           terms DF denDF  p      AIC      BIC      action
## 1           Initial model 31      3 NA 1720.891308 1823.25291 Starting model
## 2 Try dropping withinColPairs -1      0 NA  -2.281894  -5.29253      Swapped
```

It is clear in the call to `changeModelOnIC` that the model is being changed by dropping the `withinColPairs` term, which could also be achieved using `update.asreml`. However, an `asremlPlus` model-changing function operates on an `asrtests` object, that includes an `asreml` object, and, except for `changeTerms.asrtests`, results in an `asrtests` object that may contain the changed model or the supplied model depending on the results of hypothesis tests or comparisons of information criteria. In addition, the result of the test or comparison will be added to a `test.summary` data.frame stored in the new `asrtests` object and, if the model was changed, the `wald.tab` in the new `asrtests` object will have been updated for the new model.

In this case, as can be seen from the summary of `current.asrt` after the call, the model without `withinColPairs` had a smaller AIC and so now the model stored in `current.asrt` does not include `withinColPairs`. The `wald.tab` has been updated for the new model.

Choose a model for local spatial variation from several potential models

This example has been analyzed using a model for the local spatial variation based on a separable autocorrelation process of order one (Butler et al., 2020). The need for this model can be assessed using the function `addSpatialModelOnIC` from `asremlPlus` that uses a forward selection strategy for fitting a correlation model (see output below). For this function, the spatial model to be fitted, the centred covariates for the two dimensions of the grid, and the factors corresponding to the covariates must be specified. Also, `checkboundaryonly` is set to `TRUE` so that no terms are removed until the final model has been fitted and `IClikelihood` is set to `full` so that the likelihood will be based on the full REML. Because the model that incorporates the spatial model has a smaller AIC, it is the model returned in `spatial.ar1.asrt`.

The `print` of `spatial.ar1.asrt` shows that an `ar1` model for `Row` was tried first and was found to reduce the AIC by 11.898 and so became the current model. Next a model that incorporates an `ar1` function for `Column` was similarly tried and became the current model. Then an appraisal of the need for a nugget term was made by comparing the fits with the residual variance unfixed and fixed at one. The model with the unfixed residual variance was chosen and is the model to be returned. The nugget term represents non-spatial variance, such as measurement error.

```
spatial.ar1.asrt <- addSpatialModelOnIC(current.asrt, spatial.model = "corr",
                                       row.covar = "cRow", col.covar = "cColumn",
                                       row.factor = "Row", col.factor = "Column",
                                       checkboundaryonly = TRUE, ICllikelihood = "full")
```

```
## Model fitted using the gamma parameterization.
```

```
## ASReml 4.1.0 Mon Jun 12 17:43:53 2023
```

##		LogLik	Sigma2	DF	wall	cpu
## 1		-701.512	20472.38	120	17:43:53	0.0
## 2		-701.512	20472.38	120	17:43:53	0.0
## 3		-701.512	20472.38	120	17:43:53	0.0
## 4		-701.512	20472.38	120	17:43:53	0.0
## 5		-701.512	20472.38	120	17:43:53	0.0
## 6		-701.512	20472.38	120	17:43:53	0.0
## 7		-701.512	20472.38	120	17:43:53	0.0

```
## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
```

```
## Log-likelihood not converged
```

```
## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration.
```

```
## Calculating denominator DF
```

```
## Calculating denominator DF
```

```
## Model fitted using the gamma parameterization.
```

```
## ASReml 4.1.0 Mon Jun 12 17:43:54 2023
```

##		LogLik	Sigma2	DF	wall	cpu
## 1		-695.883	1877.174	120	17:43:54	0.0
## 2		-695.883	1877.102	120	17:43:54	0.0
## 3		-695.883	1876.981	120	17:43:54	0.0
## 4		-695.883	1877.039	120	17:43:54	0.0
## 5		-695.883	1877.008	120	17:43:54	0.0
## 6		-695.883	1877.024	120	17:43:54	0.0
## 7		-695.883	1877.016	120	17:43:54	0.0

```
## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
```

```
## Log-likelihood not converged
```

```
## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration.
```

```
## Calculating denominator DF
```

```
## Warning in infoCriteria.asreml(asreml.obj, ICllikelihood = ic.lik, bound.exclusions = bound.exclusion
```

```
## Column
```

```
## Warning in rmboundary.asrtests(as.asrtests(asreml.obj, wald.tab, test.summary, : In analysing yield,
## but not removed because checkboundaryonly = TRUE:
```

```
## Column
```

```
## Calculating denominator DF
```

```
## Warning in infoCriteria.asreml(new.asrtests.obj$asreml.obj, ICllikelihood = ic.lik, : The following b
```

```
## Column
```

```
## Calculating denominator DF
```

```
## Warning in infoCriteria.asreml(asrtests.obj$asreml.obj, ICllikelihood = ic.lik, : The following bound
```

```
## Column
```

```

## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:43:55 2023
##           LogLik      Sigma2      DF      wall      cpu
##  1      -671.881      4821.632     120 17:43:55     0.0
##  2      -671.881      4821.427     120 17:43:55     0.0
##  3      -671.881      4821.183     120 17:43:55     0.0
##  4      -671.881      4821.421     120 17:43:55     0.0
##  5      -671.881      4821.188     120 17:43:55     0.0
##  6      -671.881      4821.416     120 17:43:55     0.0
##  7      -671.881      4821.193     120 17:43:55     0.0

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration.

## Calculating denominator DF

## Warning in infoCriteria.asreml(asreml.obj, ICLikelihood = ic.lik, bound.exclusions = bound.exclusion
## Column, Row:Column!R

## Warning in rmboundary.asrtests(as.asrtests(asreml.obj, wald.tab, test.summary, : In analysing yield,
## but not removed because checkboundaryonly = TRUE:
## Column

## Calculating denominator DF

## Warning in infoCriteria.asreml(new.asrtests.obj$asreml.obj, ICLikelihood = ic.lik, : The following b
## Column, Row:Column!R

spatial.ar1.asrt <- rmboundary(spatial.ar1.asrt)
infoCriteria(list(nonspatial = current.asrt$asreml.obj,
                  ar1 = spatial.ar1.asrt$asreml.obj))

##           fixedDF varDF NBound      AIC      BIC      loglik
## nonspatial      0      3      0 1409.023 1417.386 -701.5117
## ar1              0      5      0 1353.762 1367.700 -671.8811

print(spatial.ar1.asrt)

##
##
## ##### Summary of the fitted variance parameters
##
##           component      std.error      z.ratio bound %ch
## Row              2.198199e+03 8.220214e+03 0.2674138      P 0.1
## Row:Column        5.182611e+04 3.379376e+04 1.5336001      P 0.0
## Row:Column!Row!cor 7.121385e-01 9.571021e-02 7.4405696      U 0.0
## Row:Column!Column!cor 8.599836e-01 1.104248e-01 7.7879542      U 0.0
## Row:Column!R        4.821195e+03 1.717266e+03 2.8074825      P 0.0
##
##
## ##### Pseudo-anova table for fixed terms
##
##
## Wald tests for fixed effects.
## Response: yield
##

```

```
##           Df denDF   F.inc    Pr
## (Intercept)  1   1.8 194.600 0.0076
## Rep         5  21.8   0.559 0.7303
## Variety     24  74.3  10.560 0.0000
##
##
## ##### Sequence of model investigations
##
## (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
##
##           terms DF denDF  p      AIC      BIC      action
## 1           Initial model 31      3 NA 1720.891308 1823.252908 Starting model
## 2 Try dropping withinColPairs -1      0 NA -2.281894 -5.292530 Swapped
## 3           Try ar1(Row) 0      2 NA -11.897888 -5.876617 Swapped
## 4           Try ar1(Column) 0      0 NA -55.397582 -55.397582 Swapped
## 5 Try fixed residual variance 0     -1 NA  1.809462 -1.201173 Unswapped
## 6           Column 1      NA NA      NA      NA      Boundary
```

However, the spatial models that are available in `asremlPlus` also include those based on two-dimensional tensor-product natural cubic smoothing splines (TPNCSS), as described by Verbyla et al. (2018), and on two-dimensional tensor-product P- splines (TPPS), as described by Rodriguez-Alvarez et al. (2018). The P-splines have been implemented using functions from the R package `TPSbits` authored by Sue Welham (2022)

The `asremlPlus` function `chooseSpatialModelOnIC` allows one to select the best model from amongst these spatial correlation models using the AIC. The four models from which it selects are (i) a separable autocorrelation model on both row and column dimensions (`corr`), (ii) a two-dimensional tensor-product natural cubic smoothing spline (TPNCSS), (ii) a two-dimensional tensor-product cubic P-spline with second-difference penalties (TPPCS), and (iii) a tensor-product two-dimensional linear P-spline with first-difference penalties (TPP1LS). By default all four are fitted and compared, but the `trySpatial` argument can be used to specify a subset of them.

The call to `chooseSpatialModelOnIC`, in addition to the arguments specifying covariates and factors, has further arguments: (i) `dropRowTerm` and `dropColTerm` that are needed in fitting P-splines, if overall Row and Column terms have been fitted in the supplied model, because the code also automatically includes these terms, (ii) an `asreml.option` argument to specify the method to be used in fitting the P-spline terms, and (iii) `return.asrts` to specify which `asrtests` objects are to be returned. Here we specify `all` so that `asrtests` objects for the fits for all four spatial models will be returned. In this case, neither the `checkboundaryonly` nor the `IClikelihood` arguments were set because their defaults for `chooseSpatialModelOnIC` are appropriate.

```
spatial.asrts <- chooseSpatialModelOnIC(current.asrt,
                                         row.covar = "cRow", col.covar = "cColumn",
                                         row.factor = "Row", col.factor = "Column",
                                         dropRowterm = "Row", dropColterm = "Column",
                                         asreml.option = "grp", return.asrts = "all")
```

```
## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration.

## Calculating denominator DF
## Calculating denominator DF
## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:43:58 2023
##           LogLik      Sigma2      DF      wall      cpu
```

```

## 1      -695.883      1877.174      120 17:43:58      0.0
## 2      -695.883      1877.102      120 17:43:58      0.0
## 3      -695.883      1876.981      120 17:43:58      0.0
## 4      -695.883      1877.039      120 17:43:58      0.0
## 5      -695.883      1877.008      120 17:43:58      0.0
## 6      -695.883      1877.024      120 17:43:58      0.0
## 7      -695.883      1877.016      120 17:43:58      0.0

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration.

## Calculating denominator DF

## Warning in infoCriteria.asreml(asreml.obj, ICLikelihood = ic.lik, bound.exclusions = bound.exclusions,
## Column

## Warning in rmboundary.asrtests(as.asrtests(asreml.obj, wald.tab, test.summary, : In analysing yield,
## but not removed because checkboundaryonly = TRUE:
## Column

## Calculating denominator DF

## Warning in infoCriteria.asreml(new.asrtests.obj$asreml.obj, ICLikelihood = ic.lik, : The following b
## Column

## Calculating denominator DF

## Warning in infoCriteria.asreml(asrtests.obj$asreml.obj, ICLikelihood = ic.lik, : The following bound
## Column

## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:43:59 2023
##      LogLik      Sigma2      DF      wall      cpu
## 1      -671.881      4821.632      120 17:43:59      0.0
## 2      -671.881      4821.427      120 17:43:59      0.0
## 3      -671.881      4821.183      120 17:43:59      0.0
## 4      -671.881      4821.421      120 17:43:59      0.0
## 5      -671.881      4821.188      120 17:43:59      0.0
## 6      -671.881      4821.416      120 17:43:59      0.0
## 7      -671.881      4821.193      120 17:43:59      0.0

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration.

## Calculating denominator DF

## Warning in infoCriteria.asreml(asreml.obj, ICLikelihood = ic.lik, bound.exclusions = bound.exclusions,
## Column, Row:Column!R

## Warning in rmboundary.asrtests(as.asrtests(asreml.obj, wald.tab, test.summary, : In analysing yield,
## but not removed because checkboundaryonly = TRUE:
## Column

## Calculating denominator DF

## Warning in infoCriteria.asreml(new.asrtests.obj$asreml.obj, ICLikelihood = ic.lik, : The following b

```



```
## Column, Row:Column!R

## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:44:00 2023
##      LogLik      Sigma2      DF      wall      cpu
## 1      -701.512      20472.38      120 17:44:01      0.0
## 2      -701.512      20472.38      120 17:44:01      0.0
## 3      -701.512      20472.38      120 17:44:01      0.0
## 4      -701.512      20472.38      120 17:44:01      0.0
## 5      -701.512      20472.38      120 17:44:01      0.0
## 6      -701.512      20472.38      120 17:44:01      0.0
## 7      -701.512      20472.38      120 17:44:01      0.0
## Spline: design points closer than 0.0009 have been merged.
## Spline: design points closer than 0.0014 have been merged.
## Spline: design points closer than 0.0009 have been merged.
## Spline: design points closer than 0.0014 have been merged.
## Calculating denominator DF

## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik, bound.exclusions = bound.exclusions)
## spl(cRow), dev(cColumn)

## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik): The following bound terms were di
## spl(cRow), dev(cColumn)

## Spline: design points closer than 0.0009 have been merged.
## Spline: design points closer than 0.0014 have been merged.

## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik): The following bound terms were di
## spl(cRow)

## Spline: design points closer than 0.0014 have been merged.
## Spline: design points closer than 0.0009 have been merged.

## Warning in changeTerms.asrtests(asrtests.obj, dropFixed = dropFixed, addFixed = addFixed, : In
## analysing yield, boundary terms removed

## Spline: design points closer than 0.0014 have been merged.
## Spline: design points closer than 0.0009 have been merged.
## Calculating denominator DF
## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:44:03 2023
##      LogLik      Sigma2      DF      wall      cpu
## 1      -701.512      20472.38      120 17:44:03      0.0
## 2      -701.512      20472.38      120 17:44:03      0.0
## 3      -701.512      20472.38      120 17:44:03      0.0
## 4      -701.512      20472.38      120 17:44:03      0.0
## 5      -701.512      20472.38      120 17:44:03      0.0
## 6      -701.512      20472.38      120 17:44:03      0.0
## 7      -701.512      20472.38      120 17:44:03      0.0
## Calculating denominator DF
## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:44:03 2023
##      LogLik      Sigma2      DF      wall      cpu
## 1      -701.512      20472.38      120 17:44:03      0.0
## 2      -701.512      20472.38      120 17:44:03      0.0
## 3      -701.512      20472.38      120 17:44:03      0.0
## 4      -701.512      20472.38      120 17:44:03      0.0
## 5      -701.512      20472.38      120 17:44:03      0.0
```

```

## 6      -701.512      20472.38      120 17:44:03      0.0
## 7      -701.512      20472.38      120 17:44:03      0.0
## Calculating denominator DF

## Warning in infoCriteria.asreml(asreml.obj, ICLikelihood = ic.lik, bound.exclusions = bound.exclusion
## grp(TP.C.1_frow), dev(cColumn))

## Warning in infoCriteria.asreml(asreml.obj, ICLikelihood = ic.lik): The following bound terms were di
## grp(TP.C.1_frow), dev(cColumn))

## Warning in infoCriteria.asreml(asreml.obj, ICLikelihood = ic.lik): The following bound terms were di
## grp(TP.C.1_frow)

## Warning in changeTerms.asrtests(asrtests.obj, dropFixed = dropFixed, addFixed = addFixed, : In
## analysing yield, boundary terms removed

## Calculating denominator DF
## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:44:07 2023
##      LogLik      Sigma2      DF      wall      cpu
## 1      -701.512      20472.38      120 17:44:07      0.0
## 2      -701.512      20472.38      120 17:44:07      0.0
## 3      -701.512      20472.38      120 17:44:07      0.0
## 4      -701.512      20472.38      120 17:44:07      0.0
## 5      -701.512      20472.38      120 17:44:07      0.0
## 6      -701.512      20472.38      120 17:44:07      0.0
## 7      -701.512      20472.38      120 17:44:07      0.0
## Calculating denominator DF
## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:44:07 2023
##      LogLik      Sigma2      DF      wall      cpu
## 1      -701.512      20472.38      120 17:44:07      0.0
## 2      -701.512      20472.38      120 17:44:07      0.0
## 3      -701.512      20472.38      120 17:44:07      0.0
## 4      -701.512      20472.38      120 17:44:07      0.0
## 5      -701.512      20472.38      120 17:44:07      0.0
## 6      -701.512      20472.38      120 17:44:07      0.0
## 7      -701.512      20472.38      120 17:44:07      0.0

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

```

```

## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Calculating denominator DF

## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik, bound.exclusions = bound.exclusion
## grp(TP.C.1_frow), dev(cRow), dev(cColumn)

## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik): The following bound terms were di
## grp(TP.C.1_frow), dev(cRow), dev(cColumn)

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik): The following bound terms were di
## grp(TP.C.1_frow), dev(cRow)

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

```

```
## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Log-likelihood not converged

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, : Some
## components changed by more than 1% on the last iteration.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in (function (fixed = y ~ 1, random = ~NULL, sparse = ~NULL, residual = ~NULL, :
## Singularities in the average information matrix.

## Warning in infoCriteria.asreml(asreml.obj, IClikelihood = ic.lik): The following bound terms were di
## grp(TP.C.1_frow)

## Warning in changeTerms.asrtests(asrtests.obj, dropFixed = dropFixed, addFixed = addFixed, : In
## analysing yield, boundary terms removed

## Calculating denominator DF
```

Output the results

```
print(spatial.asrts$spatial.IC)
```

```
##          fixedDF varDF      AIC      BIC    loglik
## nonspatial     30     3 1718.609 1817.960 -826.3047
## corr          30     5 1651.314 1756.686 -790.6570
## TPNCSS         33     6 1639.489 1756.904 -780.7445
## TPPCS          33     6 1644.190 1761.605 -783.0952
## TPP1LS         30     3 1708.443 1807.794 -821.2215
```

```
print(spatial.asrts$best.spatial.mod)
```

```
## [1] "TPNCSS"
```

```
print(spatial.asrts$asrts$TPNCSS)
```

```
##
##
## ##### Summary of the fitted variance parameters
##
##          component std.error  z.ratio bound %ch
## spl(cRow):cColumn      523.2004  372.8016 1.403429    P    0
```

```

## dev(cRow)          7664.0211 4442.0646 1.725329    P    0
## spl(cColumn)       13338.7348 9236.0511 1.444203    P    0
## spl(cColumn):cRow   366.7707 322.7484 1.136399    P    0
## spl(cRow):spl(cColumn) 3630.2187 2186.1497 1.660554    P    0
## Row:Column!R       7658.0113 1312.2026 5.835998    P    0
##
##
## ##### Pseudo-anova table for fixed terms
##
##
## Wald tests for fixed effects.
## Response: yield
##
##           Df denDF   F.inc    Pr
## (Intercept)  1   6.8 2645.00 0.0000
## Rep          5  41.3  20.45 0.0000
## Variety      24  86.5  10.15 0.0000
## cRow         1   7.0   0.07 0.7954
## cColumn       1  30.1  20.30 0.0001
## cRow:cColumn  1  64.3  22.00 0.0000
##
##
## ##### Sequence of model investigations
##
## (If a row has NA for p but not denDF, DF and denDF relate to fixed and variance parameter numbers)
##
##           terms DF denDF  p      AIC      BIC      action
## 1           Initial model 31     3 NA 1720.891308 1823.25291 Starting model
## 2 Try dropping withinColPairs -1     0 NA  -2.281894  -5.29253      Swapped
## 3      Try tensor NCS splines  3     3 NA  -79.120426 -61.05661      Swapped
## 4           dev(cColumn)  1    NA NA 1639.488959 1756.90374      Boundary
## 5           spl(cRow)  1    NA NA 1639.489011 1756.90379      Boundary
printFormulae(spatial.asrts$asrts$TPNCSS$asreml.obj)

##
##
## ##### Formulae from asreml object
##
## fixed: yield ~ Rep + Variety + cRow + cColumn + cRow:cColumn
## random: ~ spl(cColumn) + dev(cRow) + spl(cRow):cColumn + spl(cColumn):cRow + spl(cRow):spl(cColumn)
## residual: ~ Row:Column

```

The output shows that the TPNCSS model has the lowest AIC and so is selected as the best model. The model fitted for the TPNCSS model has been printed using `printFormulae.asreml`. The fitted model includes the term `dev(cRow)` that is equivalent to a random `Row` term and measures the deviations of the Row trend from a linear trend, the `spl(cRow)` term having been dropped because it was estimated to be zero. The Wald F -statistic for `Variety` is now 10.15 with 86.5 denominator degrees of freedom, as compared to 10.56 and 74.3 for the correlation model and 4.71 and 104.8 for the initial nonspatial model.

3. Diagnosing checking using residual plots and variofaces

Get current fitted asreml object and update to include standardized residuals

```
current.asr <- spatial.asrts$asrts$TPNCSS$asreml.obj
current.asr <- update(current.asr, aom=TRUE)

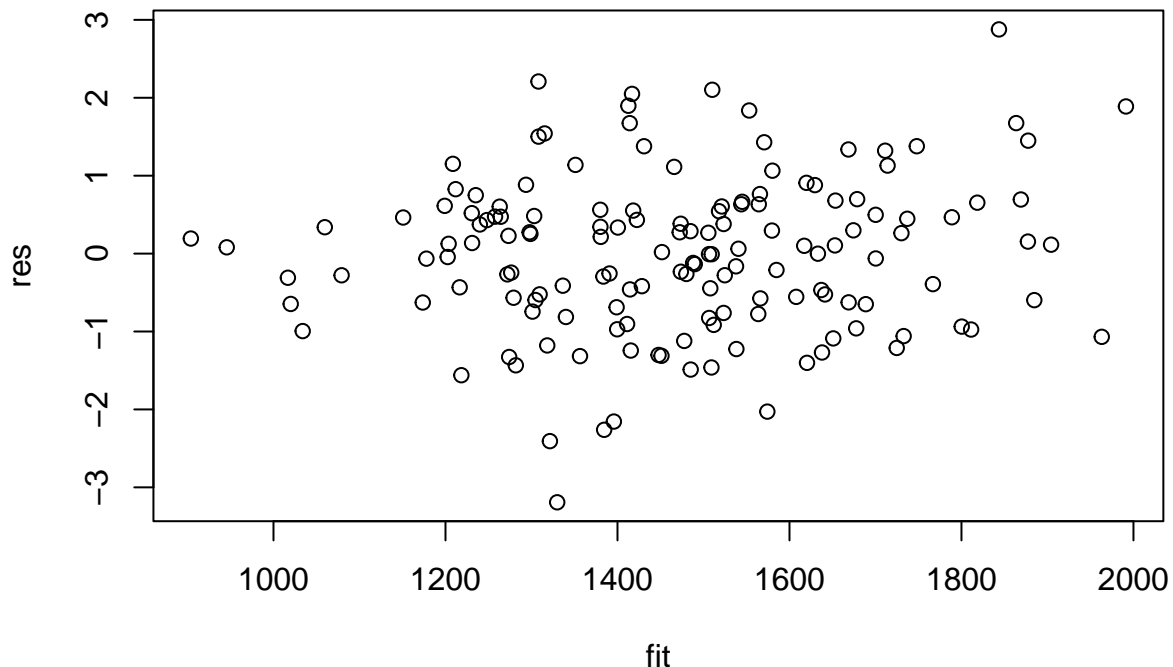
## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:44:11 2023
## Spline: design points closer than 0.0014 have been merged.
## Spline: design points closer than 0.0009 have been merged.
##      LogLik      Sigma2      DF      wall      cpu
## 1    -662.199    7657.986    117 17:44:11    0.0
## 2    -662.199    7657.984    117 17:44:11    0.0
## 3    -662.199    7657.977    117 17:44:11    0.0
## 4    -662.199    7657.975    117 17:44:11    0.0
## 5    -662.199    7657.975    117 17:44:11    0.0
## 6    -662.199    7657.974    117 17:44:11    0.0
## 7    -662.199    7657.974    117 17:44:11    0.0

Wheat.dat$res <- residuals(current.asr, type = "stdCond")
Wheat.dat$fit <- fitted(current.asr)
```

Do diagnostic checking

Do residuals-versus-fitted values plot

```
with(Wheat.dat, plot(fit, res))
```

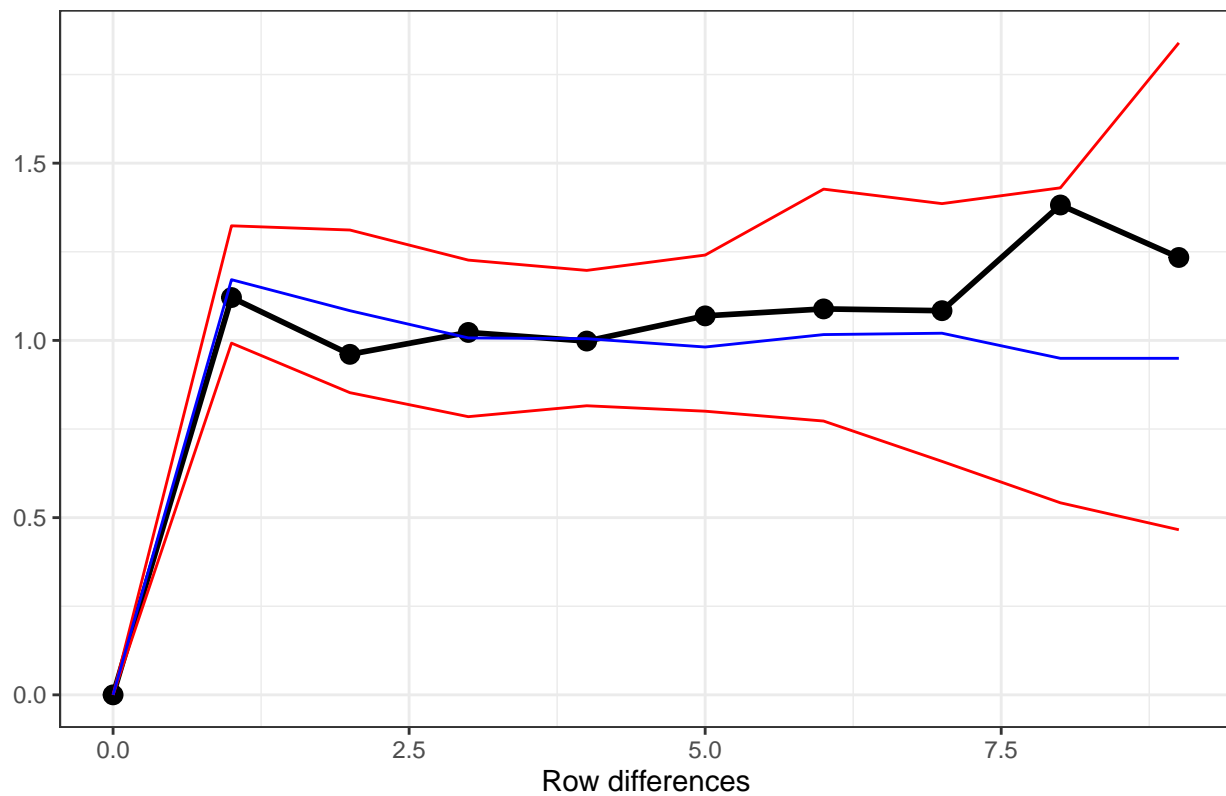


Plot variofaces

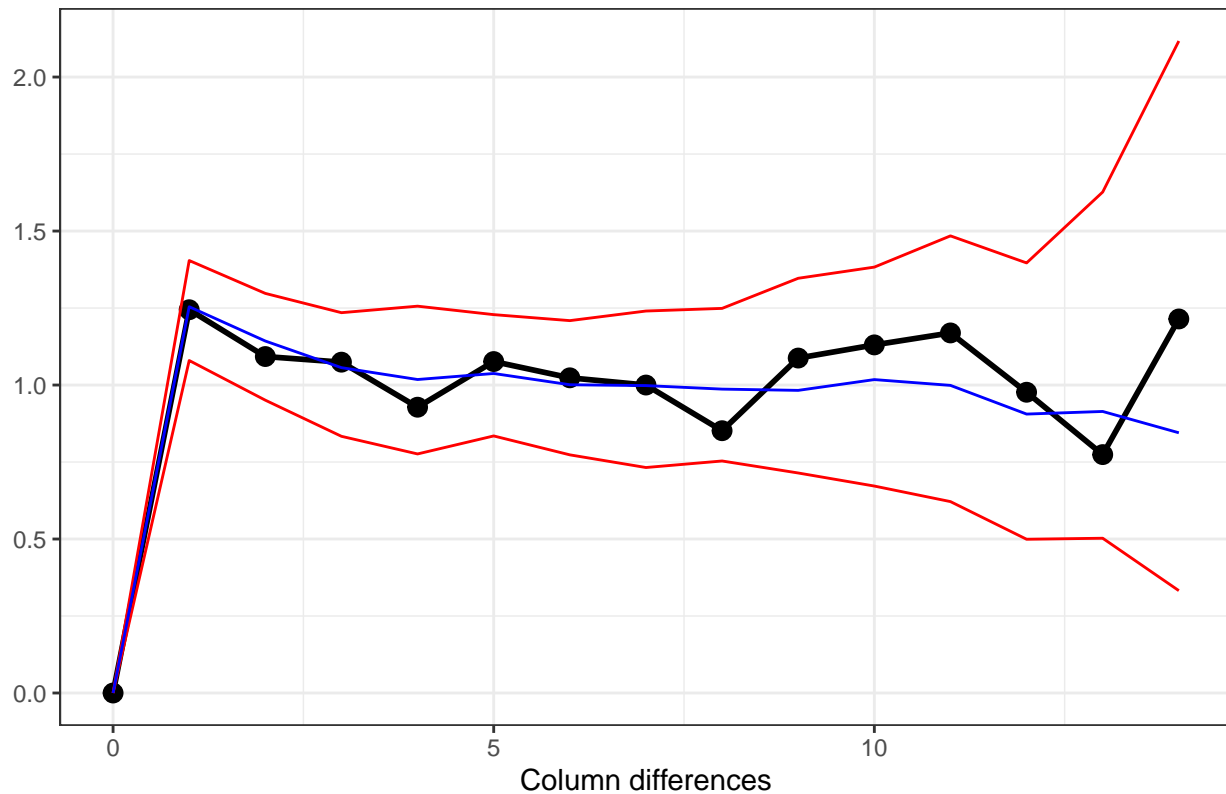
```
variofaces(current.asr, V=NULL, units="addtores",
            maxiter=50, update = FALSE)
```

```
## Model fitted using the gamma parameterization.
## ASReml 4.1.0 Mon Jun 12 17:44:14 2023
## Spline: design points closer than 0.0014 have been merged.
## Spline: design points closer than 0.0009 have been merged.
##      LogLik      Sigma2      DF      wall      cpu
## 1    -662.199    7657.974    117 17:44:14    0.0
## 2    -662.199    7657.974    117 17:44:14    0.0
## 3    -662.199    7657.974    117 17:44:14    0.0
## 4    -662.199    7657.974    117 17:44:14    0.0
## 5    -662.199    7657.974    117 17:44:14    0.0
## 6    -662.199    7657.974    117 17:44:14    0.0
## 7    -662.199    7657.974    117 17:44:14    0.0
```

Variogram face of Standardized conditional residuals for Row



Variogram face of Standardized conditional residuals for Column

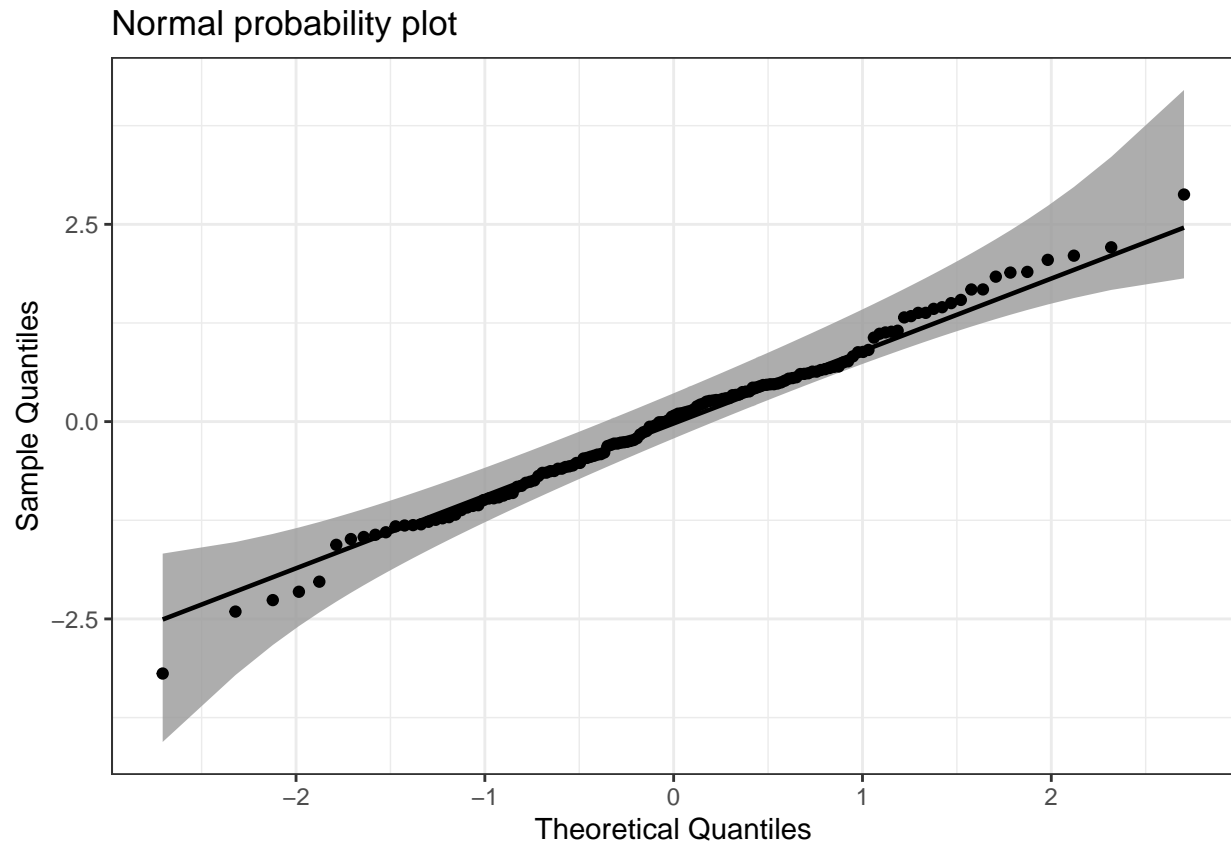


The variofaces are the lag 1 plots of the sample semivariogram with simulated confidence envelopes (Stefanova et al., 2009).

Plot normal quantile plot

The plot is obtained using the `ggplot` function with extensions available from the `qqplotr` package (Almeida et al., 2023).

```
ggplot(data = Wheat.dat, mapping = aes(sample = res)) +
  stat_qq_band(bandType = "ts") + stat_qq_line() + stat_qq_point() +
  labs(x = "Theoretical Quantiles", y = "Sample Quantiles",
       title = "Normal probability plot") +
  theme(plot.title = element_text(size = 12, face = "bold")) + theme_bw()
```

4. Prediction production and presentation

Get Variety predictions and all pairwise prediction differences and p-values

```
Var.diffs <- predictPlus(classify = "Variety",
  asreml.obj=current.asr,
  error.intervals="halfLeast",
  wald.tab=current.asrt$wald.tab,
  sortFactor = "Variety",
  tables = "predictions")

## Spline: design points closer than 0.0014 have been merged.
## Spline: design points closer than 0.0009 have been merged.
##
##
## ##### Predictions for yield from Variety
##
##
## Notes:
## - The predictions are obtained by averaging across the hypertable
##   calculated from model terms constructed solely from factors in
##   the averaging and classify sets.
## - Use 'average' to move ignored factors into the averaging set.
## - Model terms involving cRow are ignored if random and otherwise evaluated at 0.000000
## - cColumn evaluated at average value of 0.000000
## - The simple averaging set: Rep
```

```

##
##
## Variety predicted.value standard.error upper.halfLeastSignificant.limit
## 1      10      1196.214      64.11316      1255.028
## 2       9      1266.442      69.62176      1325.255
## 3      16      1268.206      74.01873      1327.020
## 4       1      1273.794      69.30592      1332.608
## 5      14      1310.806      70.75661      1369.620
## 6      23      1329.114      73.70238      1387.928
## 7      11      1340.668      75.07426      1399.481
## 8       4      1406.408      76.83336      1465.222
## 9       3      1408.543      71.85868      1467.356
## 10      7      1414.471      72.23082      1473.284
## 11     12      1423.303      71.18629      1482.117
## 12      8      1445.533      75.37805      1504.347
## 13      5      1480.687      70.31499      1539.500
## 14     15      1485.249      74.28025      1544.063
## 15     17      1495.212      71.09930      1554.026
## 16     21      1512.767      72.53109      1571.580
## 17      6      1520.841      72.50721      1579.654
## 18     24      1563.649      65.33308      1622.463
## 19     18      1568.887      71.06200      1627.700
## 20     25      1579.929      70.52509      1638.743
## 21      2      1585.398      74.96623      1644.211
## 22     22      1633.080      71.38265      1691.894
## 23     13      1637.119      66.24775      1695.932
## 24     19      1651.533      74.93686      1710.347
## 25     20      1657.612      68.65452      1716.426
## lower.halfLeastSignificant.limit est.status
## 1      1137.400 Estimable
## 2      1207.628 Estimable
## 3      1209.393 Estimable
## 4      1214.980 Estimable
## 5      1251.993 Estimable
## 6      1270.300 Estimable
## 7      1281.854 Estimable
## 8      1347.594 Estimable
## 9      1349.729 Estimable
## 10     1355.657 Estimable
## 11     1364.490 Estimable
## 12     1386.720 Estimable
## 13     1421.873 Estimable
## 14     1426.435 Estimable
## 15     1436.399 Estimable
## 16     1453.953 Estimable
## 17     1462.027 Estimable
## 18     1504.836 Estimable
## 19     1510.073 Estimable
## 20     1521.115 Estimable
## 21     1526.584 Estimable
## 22     1574.266 Estimable
## 23     1578.305 Estimable
## 24     1592.719 Estimable
## 25     1598.798 Estimable

```

```
##
##
## LSD values
##
## minimum LSD = 109.6008
##
## mean LSD = 117.6273
##
## maximum LSD = 126.3422
##
## (sed range / mean sed = 0.142 )
```

We have set `error.intervals` to `halfLeast` so that the limits for each `prediction ± (0.5 LSD)` are calculated. When these are plotted overlapping error bars indicate predictions that are not significant, while those that do not overlap are significantly different (Snee, 1981).

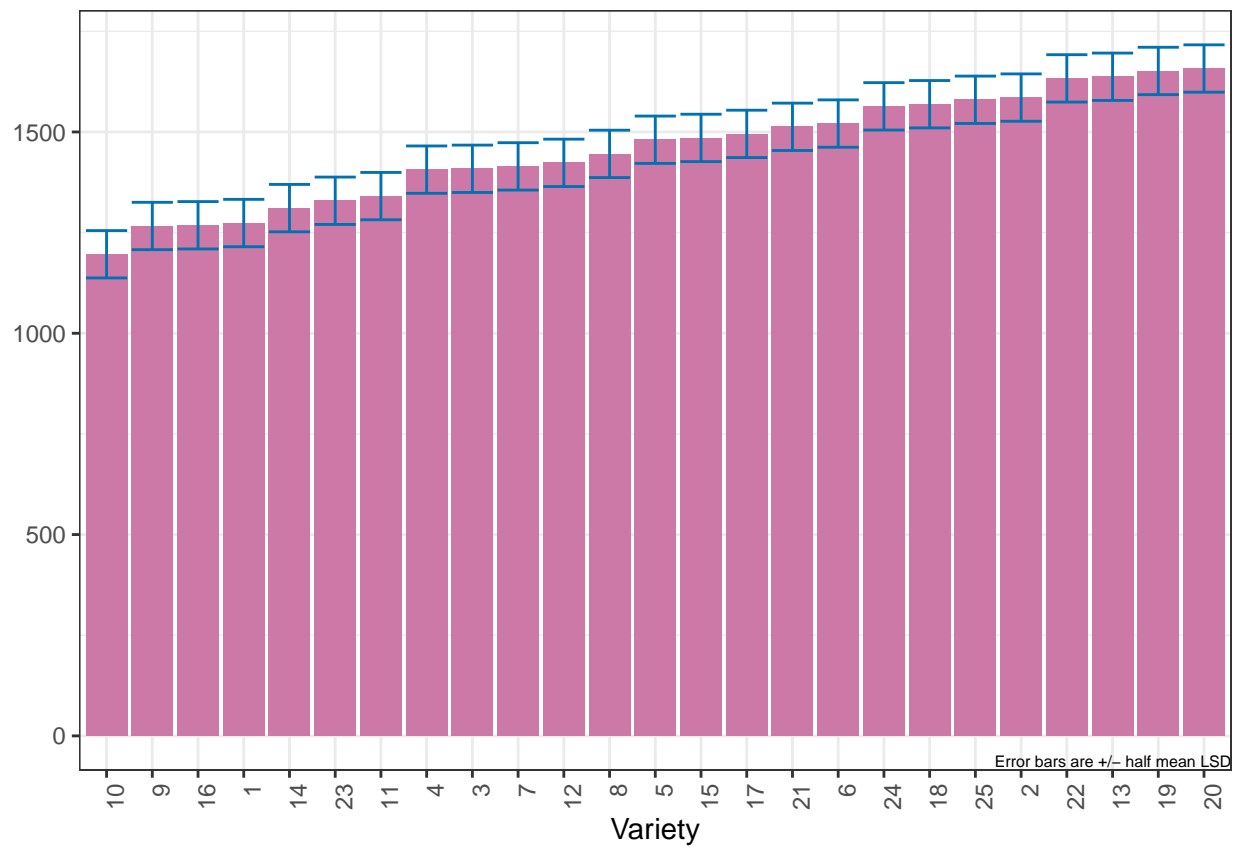
Also set was `sortFactor`, so that the results would be ordered for the values of the predictions for Variety.

The function `predictPlus` returns an `alldiffs` object, a list consisting of the following components:

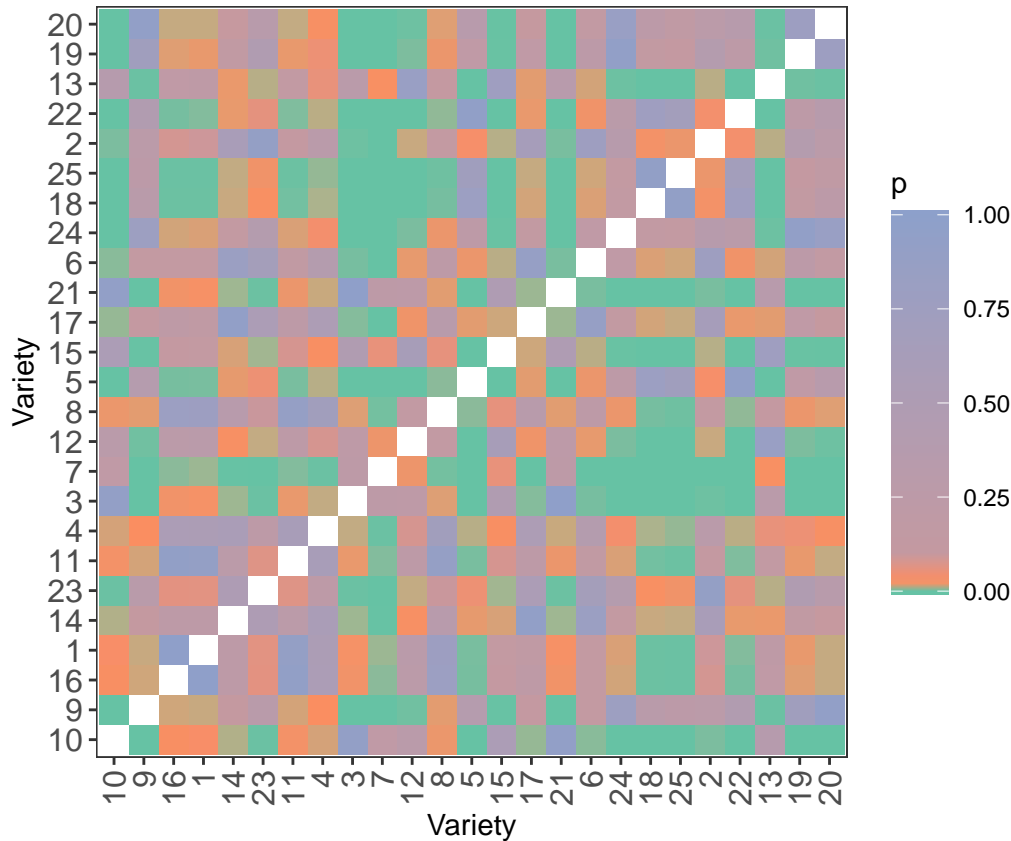
- `predictions`: the predictions, their standard errors and error intervals;
- `vcov`: the variance matrix of the predictions;
- `differences`: all pairwise differences between the predictions,
- `p.differences`: p-values for all pairwise differences between the predictions;
- `sed`: the standard errors of all pairwise differences between the predictions;
- `LSD`: the mean, minimum and maximum of the LSDs.

Plot the Variety predictions, with halfLSD intervals, and the p-values

```
plotPredictions(Var.diffs$predictions,
                 classify = "Variety", y = "predicted.value",
                 error.intervals = "half")
```



```
plotPvalues(Var.diffs)
```



References

- Almeida, A., Loy, A. and Heike Hofmann, H. (2023) **qqplotr**: *Quantile-Quantile plot extensions for ‘ggplot2’*, Version 0.0.6. <http://cran.r-project.org/package=qqplotr/> or <https://github.com/aloy/qqplotr>.
- Brien, C. J. (2023) **asremlPlus**: *Augments ASReml-R in fitting mixed models and packages generally in exploring prediction differences*. Version 4.3.53. <http://cran.r-project.org/package=asremlPlus/> or <http://chris.brien.name/rpackages/>.
- Butler, D. G., Cullis, B. R., Gilmour, A. R., Gogel, B. J. and Thompson, R. (2020). *ASReml-R Reference Manual Version 4.1.0.176*. VSN International Ltd, <https://asreml.kb.vsnr.co.uk/>.
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- Snee, R. D. (1981). Graphical Display and Assessment of Means. *Biometrics*, **37**, 835–836.
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- Stefanova, K. T., Smith, A. B. & Cullis, B. R. (2009) Enhanced diagnostics for the spatial analysis of field trials. *Journal of Agricultural, Biological, and Environmental Statistics*, **14**, 392–410.
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Experiments Requiring Residual Modelling. *Journal of Agricultural, Biological and Environmental Statistics*, **23**, 478-508.

Verbyla, A. P. (2019). A note on model selection using information criteria for general linear models estimated using REML. *Australian & New Zealand Journal of Statistics*, **61**, 39-50.

Welham, S. J. (2022) **TPSbits**: *Creates Structures to Enable Fitting and Examination of 2D Tensor-Product Splines using ASReml-R*. Version 1.0.0 <https://mmade.org/tpsbits/>