Bifactor Rotation and Reliability Coefficients

The GPArotation Package

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Bifactor models represent a factor structure where each item loads on one general factor that influences all items, plus at most one group-specific factor. They are widely used in scale development and psychometric research to investigate the relative contributions of general and specific sources of variance. This vignette illustrates how `GPArotation` can support bifactor analyses and how the results may inform decisions about total scores and subscales.

The tools presented here — bifactor rotation, omega hierarchical, and omega total — are best understood as aids to thinking rather than as decision procedures. Their interpretation always depends on substantive theory, the purpose of the measurement, and the broader research context.

Bifactor Rotation in GPArotation

Two bifactor rotation criteria are available:

- `bifactorT` — orthogonal bifactor rotation. The general factor and group factors are uncorrelated.
- `bifactorQ` — oblique bifactor rotation (biquartimin). The general factor and group factors may be correlated.

Both treat the *first column* of the loading matrix as the general factor and rotate the remaining columns as group factors. The general factor column should correspond to the dominant factor in the unrotated solution. Use `sortLoadings = FALSE` when printing to preserve the general factor in column 1 — the default sorting by variance explained may reorder the factors and obscure the bifactor structure.

```
> data("WansbeekMeijer", package = "GPArotation")
> fa.unrotated <- factanal(factors = 3, covmat = NetherlandsTV,
                          rotation = "none")
> # Orthogonal bifactor rotation
> res.bifT <- bifactorT(loadings(fa.unrotated))
> print(res.bifT, sortLoadings = FALSE, digits = 3)
```

```
Orthogonal rotation method Bifactor Biquartimin converged.
Loadings:
         Factor1 Factor2 Factor3
NL1       0.605   0.509
TV2       0.675   0.499
NL3       0.585   0.505
RTL4      0.569           0.450
RTL5      0.640           0.390
Veronica  0.644           0.489
SBS6      0.894  -0.441

               Factor1 Factor2 Factor3
SS loadings      3.111   0.968   0.596
Proportion Var   0.444   0.138   0.085
Cumulative Var   0.444   0.583   0.668
AUC              0.650   0.796   0.876
FSI              0.019   0.122   0.231
```

```
> # Oblique bifactor rotation
> res.bifQ <- bifactorQ(loadings(fa.unrotated))
> print(res.bifQ, sortLoadings = FALSE, digits = 3)
```

```
Oblique rotation method Bifactor Biquartimin converged.
Loadings:
```

```
        Factor1 Factor2 Factor3
NL1       0.640   0.462
TV2       0.708   0.453
NL3       0.619   0.460
RTL4      0.573           0.452
RTL5      0.637           0.393
Veronica  0.637           0.493
SBS6      0.861  -0.504

           Factor1 Factor2 Factor3
SS loadings   3.175   0.895   0.604
AUC           0.638   0.793   0.876
FSI           0.013   0.121   0.231

Phi:
       Factor1 Factor2 Factor3
Factor1   1.000   0.001   0.000
Factor2   0.001   1.000  -0.091
Factor3   0.000  -0.091   1.000

> # Structure matrix for oblique solution
> summary(res.bifQ, Structure = TRUE)

Oblique rotation method Bifactor Biquartimin converged in 299 iterations.
Algorithm: bb  |  fwindow: 10  |  Iterations:
Pattern (loadings) - see Structure (correlations) below:

Loadings:
        Factor1 Factor2 Factor3
NL1       0.640   0.462
TV2       0.708   0.453
NL3       0.619   0.460
RTL4      0.573           0.452
RTL5      0.637           0.393
Veronica  0.637           0.493
SBS6      0.861  -0.504

           Factor1 Factor2 Factor3
SS loadings   3.175   0.895   0.604
AUC           0.638   0.793   0.876
FSI           0.013   0.121   0.231

Correlations:
        Factor1 Factor2 Factor3
NL1       0.640   0.465
TV2       0.709   0.450
NL3       0.620   0.462
RTL4      0.573           0.445
RTL5      0.637           0.394
Veronica  0.637  -0.105   0.498
SBS6      0.861  -0.503

Hyperplane total: 7 of 14 ( 50 %) at cutoff 0.1

Post-Hoc Simplicity Suite (overall solution):
Hoffman Index:    0.467
Gini Coeficient:  0.326
Bentler Index:    0.469
```

```
Phi:
        Factor1 Factor2 Factor3
Factor1   1.000   0.001   0.000
Factor2   0.001   1.000  -0.091
Factor3   0.000  -0.091   1.000
```

The pattern matrix from `bifactorT` shows loadings on the general factor (column 1) and group factors (remaining columns). Items with high general factor loadings and near-zero group factor loadings are well represented by the total score. Items with substantial group factor loadings may be carrying domain-specific variance beyond the general factor.

For a detailed treatment of bifactor rotation see Jennrich & Bentler (2011). A comparison of exploratory bifactor analysis algorithms is provided in Garcia-Garzon et al. (2021).

Omega Hierarchical

Omega hierarchical (ω_h) quantifies how much of the total score variance may be attributable to the general factor. It is computed directly from the bifactor rotation result:

$$\omega_h = \frac{(\sum_i \lambda_{gi})^2}{(\sum_i \lambda_{gi})^2 + \sum_i \sum_j \lambda_{sij}^2 + \sum_i \theta_{ii}} \tag{1}$$

where λ_{gi} are the general factor loadings, λ_{sij} are the group factor loadings, and $\theta_{ii} = 1 - \sum_j \lambda_{ij}^2$ are the model-implied unique variances.

```
> omega_h <- function(bifactor_result) {
    L     <- loadings(bifactor_result)
    lg    <- L[, 1]
    Ls    <- L[, -1]
    theta <- 1 - rowSums(L^2)
    sum(lg)^2 / (sum(lg)^2 + sum(Ls^2) + sum(theta))
  }
> omega_t <- function(bifactor_result, R) {
    L     <- loadings(bifactor_result)
    R_hat <- L %*% t(L)
    sum(R_hat) / sum(R)
  }
> alpha_coef <- function(R) {
    k <- nrow(R)
    k / (k - 1) * (1 - sum(diag(R)) / sum(R))
  }
```

The denominator of ω_h partitions total score variance into three components: variance due to the general factor, variance due to group-specific factors, and measurement error. A higher ω_h may suggest that the general factor accounts for a larger share of total score variance, which could support the use of a total score as a summary measure — though this interpretation always depends on the substantive context.

Comparing Strong and Weak General Factors

To illustrate how ω_h varies with the relative strength of the general and group factors, we construct two population correlation matrices with known bifactor structure.

In example 1 below, the general factor is strong (loadings of 0.7) and the group factors are weak (loadings of 0.2). In Example 2 below, the general factor is weaker (loadings of 0.3) and the group factors are stronger (loadings of 0.6). Both examples use 12 items and 3 group factors of 4 items each.

```
> # Example 1: Strong general factor (loadings .7), weak group factors (.2)
> lambda_g1  <- rep(0.7, 12)
> lambda_s1a <- c(rep(0.2, 4), rep(0.0, 8))
```

```
> lambda_s1b <- c(rep(0.0, 4), rep(0.2, 4), rep(0.0, 4))
> lambda_s1c <- c(rep(0.0, 8), rep(0.2, 4))
> L1 <- cbind(lambda_g1, lambda_s1a, lambda_s1b, lambda_s1c)
> R1 <- L1 %*% t(L1)
> diag(R1) <- 1
> fa1  <- factanal(factors = 4, covmat = R1, rotation = "none")
> bif1 <- bifactorT(loadings(fa1))
> # Example 2: Weaker general factor (loadings .3), stronger group factors (.6)
> lambda_g2  <- rep(0.3, 12)
> lambda_s2a <- c(rep(0.6, 4), rep(0.0, 8))
> lambda_s2b <- c(rep(0.0, 4), rep(0.6, 4), rep(0.0, 4))
> lambda_s2c <- c(rep(0.0, 8), rep(0.6, 4))
> L2 <- cbind(lambda_g2, lambda_s2a, lambda_s2b, lambda_s2c)
> R2 <- L2 %*% t(L2)
> diag(R2) <- 1
> fa2  <- factanal(factors = 4, covmat = R2, rotation = "none")
> bif2 <- bifactorT(loadings(fa2))

> cat("Example 1 - Strong general factor:\n")

Example 1 - Strong general factor:

> cat("  alpha   =", round(alpha_coef(R1), 3), "\n")

  alpha   = 0.923

> cat("  omega_t =", round(omega_t(bif1, R1), 3), "\n")

  omega_t = 0.928

> cat("  omega_h =", round(omega_h(bif1), 3), "\n\n")

  omega_h = 0.923

> cat("Example 2 - Weaker general factor:\n")

Example 2 - Weaker general factor:

> cat("  alpha   =", round(alpha_coef(R2), 3), "\n")

  alpha   = 0.736

> cat("  omega_t =", round(omega_t(bif2, R2), 3), "\n")

  omega_t = 0.821

> cat("  omega_h =", round(omega_h(bif2), 3), "\n")

  omega_h = 0.708
```

The three coefficients address related but distinct questions:

- α estimates the proportion of total score variance that may be attributable to common factors, under the assumption of essentially tau-equivalent items (equal factor loadings). When this assumption does not hold well in practice, α may overestimate or underestimate reliability — something worth keeping in mind when interpreting it.
- ω_t relaxes the tau-equivalence assumption and estimates the proportion of total score variance that may be due to all common factors combined — general and group-specific. It may provide a more nuanced picture of reliability than α in some situations, though both are model-dependent estimates. Note that ω_t requires the observed or population correlation matrix in addition to the bifactor solution.

- ω_h focuses on the general factor only and may help address the question of how much of the total score variance could reflect the single construct the scale is primarily intended to measure. It is one piece of evidence relevant to construct validity, not a definitive answer.

The gap $\omega_t - \omega_h$ represents the proportion of total score variance that may be attributable to group-specific factors. In Example 1 this gap is small, suggesting the group factors add little beyond the general factor and the scale may be reasonably close to unidimensional. In Example 2 the gap is larger, which could suggest the group factors are contributing meaningfully and that subscale scores might be worth examining alongside the total score. Whether to act on this is a substantive judgment that goes beyond the numbers alone.

Applied Example: CCAI Climate-Friendly Purchasing Choices domain

To illustrate how these tools might inform scale interpretation in practice, we use the Climate-Friendly Purchasing Choices domain of the Climate Change Action Inventory (CCAI), analyzed by Bi and Barchard (2024). The scale has 14 items measuring the frequency of climate-friendly purchasing behaviors, with three factors identified via direct oblimin rotation: Choosing Sustainable Options, Supporting Collective Action, and Avoiding Buying New. The three factors had strong inter-correlations (0.53–0.59), which raises the question of whether a general factor might underlie all 14 items.

Bi and Barchard (2024) used `psych::principal` for component extraction with oblimin rotation, which calls `GPArotation` internally. Table 2 of the paper reports the pattern matrix from this analysis, despite being labeled "Factor Structure."

The observed 14×14 correlation matrix and published pattern matrix are included in the package as `CCAI_R` and `CCAI_pattern` respectively (see `?CCAI`). The published pattern matrix can be reproduced from the correlation matrix via eigendecomposition followed by oblimin rotation — see the examples in `?CCAI` for the complete code.

Data

```
> data("CCAI", package = "GPArotation")

> cat("Range of observed correlations:\n")

Range of observed correlations:

> cat("  Min:", round(min(CCAI_R[lower.tri(CCAI_R)]), 3), "\n")

  Min: 0.315

> cat("  Max:", round(max(CCAI_R[lower.tri(CCAI_R)]), 3), "\n")

  Max: 0.934
```

Bifactor Rotation

We extract three unrotated factors from the observed correlation matrix and apply `bifactorT`:

```
> fa_unrotated <- factanal(factors = 3, covmat = CCAI_R,
           n.obs = 461, rotation = "none")
> bif <- bifactorT(loadings(fa_unrotated))
> print(bif, sortLoadings = FALSE, digits = 3)

Orthogonal rotation method Bifactor Biquartimin converged.
Loadings:
       Factor1 Factor2 Factor3
CCAI8   0.849   0.386        
CCAI6   0.785   0.334        
CCAI7   0.760   0.346        
CCAI11  0.854           0.121
CCAI12  0.853                
CCAI10  0.788           0.177
```

```
CCAI14  0.843  -0.480
CCAI13  0.850  -0.448
CCAI5   0.773  -0.294   0.175
CCAI2   0.444           0.432
CCAI4   0.618           0.545
CCAI1   0.561           0.479
CCAI3   0.672           0.430
CCAI9   0.714           0.344

               Factor1 Factor2 Factor3
SS loadings      7.878   0.913   1.109
Proportion Var   0.563   0.065   0.079
Cumulative Var   0.563   0.628   0.707
AUC              0.615   0.856   0.857
FSI              0.007   0.117   0.121
```

The general factor (Factor 1) loadings range from 0.444 to 0.854 across all 14 items. Only one item — Item 2 ("Use borrowed/rented/digital rather than buying") — falls below 0.50, suggesting it may be the most behaviorally distinct item in the scale. The group factors contribute modest additional structure, with the Avoiding Buying New items (1, 2, 3, 4) showing the clearest group factor pattern beyond the general factor.

The variance explained by each factor may be informative: the general factor accounts for approximately 56.3% of item variance, while the two group factors account for 6.5% and 7.9% respectively, for a total of 70.7%. The dominance of the general factor relative to the group factors is consistent with the high ω_h of 0.946 calculated below.

Reliability Coefficients

```
> cat("alpha   =", round(alpha_coef(CCAI_R), 3), "\n")

alpha   = 0.95

> cat("omega_t =", round(omega_t(bif, CCAI_R), 3), "\n")

omega_t = 0.965

> cat("omega_h =", round(omega_h(bif), 3), "\n")

omega_h = 0.946

> cat("gap (omega_t - omega_h) =",
      round(omega_t(bif, CCAI_R) - omega_h(bif), 3), "\n")

gap (omega_t - omega_h) = 0.019
```

All three coefficients are high and close to each other. The small gap between ω_t and ω_h (≈ 0.019) could suggest that the group factors contribute little unique variance beyond the general factor.

Partitioning Total Score Variance

```
> omega_h_by_group <- function(bifactor_result, R) {
    L     <- loadings(bifactor_result)
    lg    <- L[, 1]
    Ls    <- L[, -1]
    theta <- 1 - rowSums(L^2)
    denom <- sum(lg)^2 + sum(Ls^2) + sum(theta)
    cat("Variance partition:\n")
    cat("  General factor:    ", round(sum(lg)^2 / denom, 3), "\n")
    for (j in 1:ncol(Ls))
      cat("  Group factor", j + 1, ":     ",
          round(sum(Ls[, j]^2) / denom, 3), "\n")
    cat("  Measurement error: ", round(sum(theta) / denom, 3), "\n")
```

```
    cat("  Total (omega_t):   ",
        round(omega_t(bifactor_result, R), 3), "\n")
  }

> omega_h_by_group(bif, CCAI_R)

Variance partition:
  General factor:     0.946
  Group factor 2 :      0.008
  Group factor 3 :      0.01
  Measurement error:  0.036
  Total (omega_t):    0.965
```

The partition may be informative here. Approximately 94.6% of total score variance could be attributable to the general factor, with the two group factors together contributing around 1.8% and measurement error around 3.6%. This pattern could suggest that the 14-item total score is capturing a single dominant construct — something like general climate-conscious purchasing behavior — with the three subscales adding relatively little unique psychometric information beyond that.

What This May Mean

Bi and Barchard (2024) identified three theoretically meaningful subscales and noted their strong inter-correlations. The bifactor analysis here offers one additional perspective: the subscales may be useful for targeting specific interventions (for example, campaigns to encourage buying used, or to support collective action), but from a psychometric standpoint the total score may capture most of the reliable variance in climate-friendly purchasing behavior.

This does not mean the subscales are without value — substantive and practical considerations matter as much as psychometric ones. A researcher interested in behavior change might find the subscale distinctions more actionable than a total score, regardless of what the reliability coefficients suggest. These analyses are tools for thinking through the structure of a scale, not verdicts on how it should be used.

Practical Considerations

- The general factor must be in the first column of the bifactor solution. Use `sortLoadings = FALSE` when printing to preserve this ordering.
- ω_h uses model-implied unique variances ($\theta_{ii} = 1 - \sum_j \lambda_{ij}^2$). If the observed correlation matrix is available, unique variances can be estimated as $\theta_{ii} = R_{ii} - \hat{R}_{ii}$ where $\hat{R} = LL^T$.
- $\omega_h > 0.80$ has been suggested as supporting total score interpretation as a reasonably unidimensional construct (Reise, 2012), though this threshold should not be applied mechanically.
- $\omega_h < 0.60$ may suggest that subscale scores are worth examining alongside the total score, depending on the context.
- Values between 0.60 and 0.80 require judgment — examine the general factor loadings for uniformity and consider the theoretical rationale for the subscales.
- These coefficients are tools for thinking, not oracles. They may raise useful questions and provide one perspective on scale structure, but their interpretation always depends on the specific context, the theoretical framework, and the intended purpose of the measurement.

Further Resources

For detailed treatment of bifactor models and their applications see Jennrich & Bentler (2011) and Mansolf & Reise (2016). For a comparison of exploratory bifactor analysis algorithms see Garcia-Garzon et al. (2021). For background on omega coefficients and their interpretation see Reise (2012).

The `psych` package provides additional tools for bifactor analysis and reliability estimation, including `omega()` for computing omega coefficients directly from a correlation matrix.

References

Bi, Y. and Barchard, K.A. (2024). Purchasing choices that reduce climate change: An exploratory factor analysis. *Spectra Undergraduate Research Journal*, **3**(2), 8–14. doi: 10.9741/2766-7227.1028

Garcia-Garzon, E., Abad, F.J., and Garrido, L.E. (2021). On omega hierarchical estimation: A comparison of exploratory bi-factor analysis algorithms. *Multivariate Behavioral Research*, **56**(1), 101–119. doi: 10.1080/00273171.2020.1736977

Jennrich, R.I. and Bentler, P.M. (2011). Exploratory bi-factor analysis. *Psychometrika*, **76**(4), 537–549. doi: 10.1007/s11336-011-9218-4

Mansolf, M. and Reise, S.P. (2016). Exploratory bifactor analysis: The Schmid-Leiman orthogonalization and Jennrich-Bentler analytic rotations. *Multivariate Behavioral Research*, **51**(5), 698–717. doi: 10.1080/00273171.2016.1215898

Reise, S.P. (2012). The rediscovery of bifactor measurement models. *Multivariate Behavioral Research*, **47**(5), 667–696. doi: 10.1080/00273171.2012.715555