Package ‘CircNNTSRSymmetric’

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Type Package

Title Circular Data using Symmetric NNTS Models

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Description The statistical analysis of circular data using distributions based on symmetric Nonnegative Trigonometric Sums (NNTS). It includes functions to perform empirical analysis and estimate the parameters of density functions. Fernandez-Duran, J.J. and Gregorio-Dominguez, M.M. (2025), ``Multimodal Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums and a Likelihood Ratio Test for Reflective Symmetry'', <doi:10.48550/arXiv.2412.19501>.

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`CircNNTSRSymmetric-package`

CircNNTSRSymmetric: An R Package for the statistical analysis of circular data using symmetric nonnegative trigonometric sums (NNTS) models. Fernández-Durán, J.J., Gregorio-Domínguez, M.M. (2025). Multimodal Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums and a Likelihood Ratio Test for Reflective Symmetry, arXiv:2412.19501 [stat.ME] (available at https://arxiv.org/abs/2412.19501)

Description

The statistical analysis of circular data using distributions based on symmetric Nonnegative Trigonometric Sums (NNTS). It includes functions to perform empirical analysis and estimate the parameters of density functions. Fernández-Durán, J.J. and Gregorio-Domínguez, M.M. (2025) <doi:10.48550/arXiv.2412.19501>.

Details

Package:	CircNNTSRSymmetric
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Version:	0.1.0
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The NNTS (Non-Negative Trigonometric Sums) symmetric density around μ is defined as:

$$f(\theta; M, \underline{c}, \mu) = \sum_{k=0}^{M}\sum_{l=0}^{M} \rho_k \rho_l e^{i(k-l)(\theta-\mu)}$$

with ρ_k real numbers for $k = 0, \ldots, M$ with $\sum_{k=0}^{M} \rho_k^2 = \frac{1}{2\pi}$.

Equivalently, the symmetric NNTS density is:

$$f(\theta; M, \underline{c}, \mu) = \frac{1}{2\pi}\sum_{k=0}^{M}\sum_{l=0}^{M} ||c_k||||\bar{c}_l||e^{i(k-l)(\theta-\mu)} = \frac{1}{2\pi}\sum_{k=0}^{M}\sum_{l=0}^{M} c_{Sk}\bar{c}_{Sl}e^{i(k-l)\theta}$$

. The parameters $c_{Sk} = ||c_k||e^{-ik\mu}$ are the parameters of the general (non-symmetric) NNTS model.

The symmetric NNTS model is derived from the general NNTS model (Fernández-Durán, 2004 and Fernández-Durán and Gregorio-Domínguez, 2016) with norms (moduli) of the c parameters equal in both models and arguments of the c parameters equal to $\phi_k = -k\mu$ for $k = 1, 2, \ldots, M$.

Author(s)

Juan Jose Fernandez-Duran and Maria Mercedes Gregorio-Dominguez

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References

Fernández-Durán, J.J. (2004). Circular Distributions Based on Nonnegative Trigonometric Sums. *Biometrics*, 60, pp. 499-503.

Fernández-Durán, J.J. and Gregorio-Domínguez, M.M. (2016). CircNNTSR: An R Package for the Statistical Analysis of Circular, Multivariate Circular, and Spherical Data Using Nonnegative Trigonometric Sums. *Journal of Statistical Software*, 70(6), 1-19. doi:10.18637/jss.v070.i06

Fernández-Durán, J.J., Gregorio-Domínguez, M.M. (2025). Multimodal Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums and a Likelihood Ratio Test for Reflective Symmetry, arXiv:2412.19501 [stat.ME] (available at https://arxiv.org/abs/2412.19501)

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nntsmanifoldnewtonestimationgradientstop

Parameter estimation for NNTS distributions with gradient stop

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Description

Computes the maximum likelihood estimates of the NNTS parameters of an NNTS distribution, using a Newton algorithm on the hypersphere and considering a maximum number of iterations determined by a constraint in terms of the norm of the gradient

Usage

```
nntsmanifoldnewtonestimationgradientstop(data, M = 0, iter = 1000,
initialpoint = FALSE, cinitial,gradientstop=1e-10)
```

Arguments

`data`	Vector of angles in radians
`M`	Number of components in the NNTS symmetric density
`iter`	Number of iterations
`initialpoint`	TRUE if an initial point for the optimization algorithm for the general (asymmetric) NNTS density will be used
`cinitial`	Vector of size M+1. The first element is real and the next M elements are complex (values for c_0 and $c_1, ...,c_M$). The sum of the squared moduli of the parameters must be equal to 1/(2*pi). This is the vector of parameters for the general (asymmetric) NNTS density
gradientstop	
`gradientstop`	The minimum value of the norm of the gradient to stop the Newton algorithm on the hypersphere

Value

`cestimates`	Matrix of (M+1)x2. The first column is the parameter numbers, and the second column is the c parameter's estimators of the NNTS model
`loglik`	Optimum log-likelihood value for the NNTS model
`AIC`	Value of Akaike's Information Criterion
`BIC`	Value of Bayesian Information Criterion
`gradnormerror`	Gradient error after the last iteration

Author(s)

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

References

Fernández-Durán, J.J., Gregorio-Domínguez, M.M. (2025). Multimodal Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums and a Likelihood Ratio Test for Reflective Symmetry, arXiv:2412.19501 [stat.ME] (available at https://arxiv.org/abs/2412.19501)

Examples

```
data(Turtles_radians)
resturtles<-nntsmanifoldnewtonestimationgradientstop(data=Turtles_radians, M = 2,
iter=1000,gradientstop=1e-10)
resturtles
```

---

nntsmanifoldnewtonestimationsymmetry

Parameter estimation for NNTS symmetric distributions

Description

Computes the maximum likelihood estimates of the NNTS parameters of an NNTS symmetric distribution, using a Newton algorithm on the hypersphere

Usage

```
nntsmanifoldnewtonestimationsymmetry(data, M = 0,iter=1000,gradientstop=1e-10,
pevalmu=1000,initialpoint=FALSE,cinitial)
```

Arguments

`data`	Vector of angles in radians
`M`	Number of components in the NNTS symmetric density
`iter`	Number of iterations
`gradientstop`	The minimum value of the norm of the gradient to stop the Newton algorithm on the hypersphere

`pevalmu` Number of equidistant points in the interval 0 to 2pi to search for the maxima of the angle of symmetry

`initialpoint` TRUE if an initial point for the optimization algorithm for the general (asymmetric) NNTS density will be used

`cinitial` Vector of size M+1. The first element is real and the next M elements are complex (values for c_0 and $c_1, ...,c_M$). The sum of the squared moduli of the parameters must be equal to 1/(2*pi). This is the vector of parameters for the general (asymmetric) NNTS density

Value

`cestimatessym` Matrix of (M+1)x2. The first column is the parameter numbers, and the second column is the c parameter's estimators of the symmetric NNTS model

`mu` Estimate of the angle of symmetry of the NNTS symmetric model

`logliksym` Optimum log-likelihood value for the NNTS symmetric model

`AICsym` Value of Akaike's Information Criterion for the NNTS symmetric model

`BICsym` Value of Bayesian Information Criterion for the NNTS symmetric model

`gradnormerrorsym`
Gradient error after the last iteration for the estimation of the parameters of the NNTS symmetric model

`cestimatesnonsym`
Matrix of (M+1)x2. The first column is the parameter numbers, and the second column is the c parameter's estimators of the symmetric NNTS model

`logliknonsym` Optimum log-likelihood value for the general (non-symmetric) NNTS model

`AICnonsym` Value of Akaike's Information Criterion for the general (non-symmetric) NNTS model

`BICnonsym` Value of Bayesian Information Criterion for the general (non-symmetric) NNTS model

`gradnormerrornonsym`
Gradient error after the last iteration for the estimation of the parameters of the general (non-symmetric) NNTS model

`loglikratioforsym`
Value of the likelihood ratio test statistic for symmetry

`loglikratioforsympvalue`
Value of the asymptotic chi squared p-value of the likelihood ratio test statistic for symmetry

Author(s)

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

References

Fernández-Durán, J.J., Gregorio-Domínguez, M.M. (2025). Multimodal Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums and a Likelihood Ratio Test for Reflective Symmetry, arXiv:2412.19501 [stat.ME] (available at https://arxiv.org/abs/2412.19501)

Examples

```
data(Turtles_radians)
resturtlessymm<-nntsmanifoldnewtonestimationsymmetry(data=Turtles_radians, M = 2, iter =1000,
gradientstop=1e-10,pevalmu=1000)
resturtlessymm
hist(Turtles_radians,breaks=seq(0,2*pi,2*pi/13),xlab="Direction (radians)",freq=FALSE,
ylab="",main="",ylim=c(0,.8),axes=FALSE)
nntsplot(resturtlessymm$cestimatessym[,2],2,add=TRUE)
nntsplot(resturtlessymm$cestimatesnonsym[,2],2,add=TRUE,lty=2)
axis(1,at=c(0,pi/2,pi,6*(pi/4),2*pi),labels=c("0",expression(pi/2),expression(pi),
expression(3*pi/2),expression(2*pi)),las=1)
axis(2)

data(Ants_radians)
resantssymm<-nntsmanifoldnewtonestimationsymmetry(data=Ants_radians, M = 4, iter =1000,
gradientstop=1e-10,pevalmu=1000)
resantssymm
hist(Ants_radians,breaks=seq(0,2*pi,2*pi/13),xlab="Direction (radians)",freq=FALSE,
ylab="",main="",ylim=c(0,.8),axes=FALSE)
nntsplot(resantssymm$cestimatessym[,2],4,add=TRUE)
nntsplot(resantssymm$cestimatesnonsym[,2],4,add=TRUE,lty=2)
axis(1,at=c(0,pi/2,pi,6*(pi/4),2*pi),labels=c("0",expression(pi/2),expression(pi),
expression(3*pi/2),expression(2*pi)),las=1)
axis(2)
```

---

nntsmanifoldnewtonestimationsymmetryknownsymmetryanglemu

Parameter estimation for NNTS symmetric distributions

Description

Computes the maximum likelihood estimates of the NNTS parameters of an NNTS symmetric distribution with known angle of symmetry mu, using a Newton algorithm on the hypersphere

Usage

```
nntsmanifoldnewtonestimationsymmetryknownsymmetryanglemu(data, mu, M = 0,
iter=1000,gradientstop=1e-10,initialpoint=FALSE,cinitial)
```

Arguments

`data`	Vector of angles in radians
`mu`	Known angle of symmetry of the NNTS symmetric model
`M`	Number of components in the NNTS symmetric density
`iter`	Number of iterations
`gradientstop`	The minimum value of the norm of the gradient to stop the Newton algorithm on the hypersphere

`initialpoint` TRUE if an initial point for the optimization algorithm for the general (asymmetric) NNTS density will be used

`cinitial` Vector of size M+1. The first element is real and the next M elements are complex (values for c_0 and $c_1, ...,c_M$). The sum of the squared moduli of the parameters must be equal to 1/(2*pi). This is the vector of parameters for the general (asymmetric) NNTS density

Value

`cestimatessym` Matrix of (M+1)x2. The first column is the parameter numbers, and the second column is the c parameter's estimators of the symmetric NNTS model

`mu` Known angle of symmetry of the NNTS symmetric model

`logliksym` Optimum log-likelihood value for the NNTS symmetric model

`AICsym` Value of Akaike's Information Criterion for the NNTS symmetric model

`BICsym` Value of Bayesian Information Criterion for the NNTS symmetric model

`gradnormerrorsym`
Gradient error after the last iteration for the estimation of the parameters of the NNTS symmetric model

`cestimatesnonsym`
Matrix of (M+1)x2. The first column is the parameter numbers, and the second column is the c parameter's estimators of the symmetric NNTS model

`logliknonsym` Optimum log-likelihood value for the general (non-symmetric) NNTS model

`AICnonsym` Value of Akaike's Information Criterion for the general (non-symmetric) NNTS model

`BICnonsym` Value of Bayesian Information Criterion for the general (non-symmetric) NNTS model

`gradnormerrornonsym`
Gradient error after the last iteration for the estimation of the parameters of the general (non-symmetric) NNTS model

`loglikratioforsym`
Value of the likelihood ratio test statistic for symmetry

`loglikratioforsympvalue`
Value of the asymptotic chi squared p-value of the likelihood ratio test statistic for symmetry

Author(s)

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

References

Fernández-Durán, J.J., Gregorio-Domínguez, M.M. (2025). Multimodal Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums and a Likelihood Ratio Test for Reflective Symmetry, arXiv:2412.19501 [stat.ME] (available at https://arxiv.org/abs/2412.19501)

Examples

```
data(Ants_radians)
resantssymmknownmu<-nntsmanifoldnewtonestimationsymmetryknownsymmetryanglemu(data=Ants_radians,
mu=pi, M = 4, iter =1000,gradientstop=1e-10)
resantssymmknownmu
hist(Ants_radians,breaks=seq(0,2*pi,2*pi/13),xlab="Direction (radians)",freq=FALSE,
ylab="",main="",ylim=c(0,.8),axes=FALSE)
nntsplot(resantssymmknownmu$cestimatessym[,2],4,add=TRUE)
nntsplot(resantssymmknownmu$cestimatesnonsym[,2],4,add=TRUE,lty=2)
axis(1,at=c(0,pi/2,pi,6*(pi/4),2*pi),labels=c("0",expression(pi/2),expression(pi),
expression(3*pi/2),expression(2*pi)),las=1)
axis(2)
```

---

nntsmeasureslocationdispersion

Moments of an NNTS density

Description

Computes the first moment, second moment, mean direction, dispersion, circular varance, coefficient of asymmetry and kurtosis from the given parameters of an NNTS density.

Usage

```
nntsmeasureslocationdispersion(cestimates,M=0)
```

Arguments

`cestimates`	Matrix of (M+1)x2. The first column is the parameter numbers, and the second column is the c parameter vecto (or c estimates) of the NNTS model
`M`	Number of components in the NNTS density

Value

`firstmoment`	Value of the first trigonometric moment
`secondmoment`	Value of the second trigonometric moment
`meandirection`	Value of the mean direction
`dispersion`	Value of the dispersion
`circularvariance`	Value of the circular variance
`asymmetrycoefficient`	Value of the coefficient of asymmetry
`kurtosis`	Value of the kurtosis

Author(s)

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

References

Fernández-Durán, J.J., Gregorio-Domínguez, M.M. (2025). Multimodal Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums and a Likelihood Ratio Test for Reflective Symmetry, arXiv:2412.19501 [stat.ME] (available at https://arxiv.org/abs/2412.19501)

Examples

```
data(Ants_radians)
resants<-nntsmanifoldnewtonestimationgradientstop(data=Ants_radians, M = 2, iter=1000,
gradientstop=1e-10)
resants
nntsmeasureslocationdispersion(resants$cestimates,M=2)
```

`samplecircularskewness` *Calculation of the Sample Skewness*

Description

Computes the skewness for a sample of angles

Usage

```
samplecircularskewness(data)
```

Arguments

`data`	Vector of angles in radians

Value

`Value`	Value of the sample skewness

Author(s)

Juan Jose Fernandez-Duran y Maria Mercedes Gregorio-Dominguez

References

Fernández-Durán, J.J., Gregorio-Domínguez, M.M. (2025). Multimodal Symmetric Circular Distributions Based on Nonnegative Trigonometric Sums and a Likelihood Ratio Test for Reflective Symmetry, arXiv:2412.19501 [stat.ME] (available at https://arxiv.org/abs/2412.19501)

Examples

```
data(Ants_radians)
samplecircularskewness(data=Ants_radians)

# non-symmetric
cp3a<-c(0.27672975+0.00000000i,-0.04547516-0.00298663i,-0.18680096-0.10457410i,
0.03339396-0.183317526i)
cp3a<-cp3a/sqrt(sum(Mod(cp3a)^2))
cp3a<-(1/sqrt(2*pi))*cp3a

cp3annts<-cbind(c(0,1,2,3),cp3a)
nntsmeasureslocationdispersion(cp3annts,M=3)
set.seed(1234567890)
datasim3a<-nntssimulation(1000,cp3a,3)$simulations
samplecircularskewness(datasim3a)

#symmetric
cp3b<-Mod(cp3a)
cp3bnnts<-cbind(c(0,1,2,3),cp3b)
nntsmeasureslocationdispersion(cp3bnnts,M=3)
set.seed(1234567890)
datasim3b<-nntssimulation(1000,cp3b,3)$simulations
samplecircularskewness(datasim3b)

#symmetric bis
cp3c<-c(0.3131489,0.1421822,0.1266749,0.1575766)
cp3c<-cp3c/sqrt(sum(Mod(cp3c)^2))
cp3c<-(1/sqrt(2*pi))*cp3c
cp3c<-cp3c*exp((0:3)*1i*(-pi))
cp3cnnts<-cbind(c(0,1,2,3),cp3c)
nntsmeasureslocationdispersion(cp3cnnts,M=3)
set.seed(1234567890)
datasim3c<-nntssimulation(1000,cp3c,3)$simulations
samplecircularskewness(datasim3c)
```

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