Package 'KEPTED'

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Type Package

Title Kernel-Embedding-of-Probability Test for Elliptical Distribution

URL https://github.com/tyy20/KEPTED

BugReports https://github.com/tyy20/KEPTED/issues

Version 0.2.0

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Description Provides an implementation of a kernel-embedding of probability test for elliptical distribution. This is an asymptotic test for elliptical distribution under general alternatives, and the location and shape parameters are assumed to be unknown. Some side-products are posted, including the transformation between rectangular and polar coordinates and two product-type kernel functions. See Tang and Li (2024) <doi:10.48550/arXiv.2306.10594> for details.

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R topics documented:

`EllKEPT` *Kernel embedding of probability test for elliptical distribution*

Description

This function gives a test on whether the data is elliptically distributed based on kernel embedding of probability. See Tang and Li (2024) for details. Gaussian kernels and product-type inverse quadratic kernels are considered.

Usage

```
EllKEPT(
  X,
  eps = 1e-06,
  kerU = "Gaussian",
  kerTheta = "Gaussian",
  gamma.U = 0,
  gamma.Theta = 0
)
```

Arguments

`X`	A matrix with n rows and d columns.
`eps`	The regularization constant added to the diagonal to avoid singularity. Default value is `1e-6`.
`kerU`	The type of kernel function on `U`. Currently supported options are `"Gaussian"` and `"PIQ"`.
`kerTheta`	The type of kernel function on `Theta`. Currently supported options are `"Gaussian"` and `"PIQ"`.
`gamma.U`	The tuning parameter `gamma` in the kernel function k_U(u1,u2). If `gamma.U=0`, the recommended procedure of selecting tuning parameter will be applied. Otherwise, the value given in `gamma.U` will be directly used as the tuning parameter. Default value is `gamma.U=0`. See "Details" for more information.
`gamma.Theta`	The tuning parameter `gamma` in the kernel function k_Theta(theta1,theta2). If `gamma.Theta=0`, the recommended procedure of selecting tuning parameter will be applied. Otherwise, the value given in `gamma.Theta` will be directly used as the tuning parameter. Default value is `gamma.Theta=0`. See "Details" for more information.

Details

The Gaussian kernel is defined as k(z1,z2)=exp(-gamma*||z1-z2||^2), and the Product-type Inverse-Quadratic (`PIQ`) kernel is defines as k(z1,z2)=Prod_j(1/(1+gamma*(z1_j-z2_j)^2)). The recommended procedure of selecting tuning parameter is given as in the simulation section of Tang and Li (2023+), where we set 1/sqrt(gamma)=(n(n-1)/2)^(-1)*sum_{1<=i<j<=n}||Z_i-Z_j||.

Value

A list of the following:

`stat`	The value of the test statistic.
`pval`	The p-value of the test.
`lambda`	The `n` eigenvalues in the approximated asymptotic distribution.
`gamma.U`	The tuning parameter `gamma.U` used in the test. Same as the input if its input is nonzero.
`gamma.Theta`	The tuning parameter `gamma.Theta` used in the test. Same as the input if its input is nonzero.

Note

In the arguments, `eps` refers to a regularization constant added to the diagonal. When the dimension is high, we recommend increasing `eps` to avoid singularity.

References

Tang, Y. and Li, B. (2024), "A nonparametric test for elliptical distribution based on kernel embedding of probabilities," https://arxiv.org/abs/2306.10594

Examples

```
set.seed(313)
n=50
d=3

## Null Hypothesis
X=matrix(rnorm(n*d),nrow=n,ncol=d)
EllKEPT(X)

## Alternative Hypothesis
X=matrix(rchisq(n*d,2)-2,nrow=n,ncol=d)
EllKEPT(X)
```

`KEPTED`	*Provides an implementation of a kernel-embedding of probability test for elliptical distribution. This is an asymptotic test for elliptical distribution under general alternatives, and the location and shape parameters are assumed to be unknown. Some side-products are posted, including the transformation between rectangular and polar coordinates and two product-type kernel functions.*

Description

Provides an implementation of a kernel-embedding of probability test for elliptical distribution. This is an asymptotic test for elliptical distribution under general alternatives, and the location and shape parameters are assumed to be unknown. Some side-products are posted, including the transformation between rectangular and polar coordinates and two product-type kernel functions.

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References

Tang, Y. and Li, B. (2024), "A nonparametric test for elliptical distribution based on kernel embedding of probabilities," https://arxiv.org/abs/2306.10594

`kerGauss`	*Gaussian kernel computation*

Description

Computing the values of Gaussian kernel functions.

Usage

```
kerGauss(gamma, z1, z2)
```

Arguments

`gamma`	A number, the bandwidth parameter in the Gaussian kernel.
`z1`	A vector, the first input of the Gaussian kernel.
`z2`	A vector, the second input of the Gaussian kernel.

Details

The Gaussian kernel is defined as k(z1,z2)=exp(-gamma*||z1-z2||^2).

Value

A number, the value of the Gaussian kernel function.

Examples

```
gamma=0.02
z1=c(3,1,3)
z2=c(8,1,9)
kerGauss(gamma,z1,z2)
```

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kerPIQ — *Product-type Inverse-Quadratic (PIQ) kernel computation*

Description

Computing the values of Product-type Inverse-Quadratic (PIQ) kernel functions.

Usage

```
kerPIQ(gamma, z1, z2)
```

Arguments

gamma	A number, the bandwidth parameter in the PIQ kernel.
z1	A vector, the first input of the PIQ kernel.
z2	A vector, the second input of the PIQ kernel.

Details

The Product-type Inverse-Quadratic (PIQ) kernel is defined as k(z1,z2)=Prod_j(1/(1+gamma*(z1_j-z2_j)^2)).

Value

A number, the value of the PIQ kernel function.

Examples

```
gamma=0.02
z1=c(3,1,3)
z2=c(8,1,9)
kerPIQ(gamma,z1,z2)
```

Polar2Rec *Polar to rectangular coordinates*

Description

Given a polar coordinate representation `(R,Theta)` of a d-dimensional vector `X`, where `R` is the length of `X` and the `(d-1)`-dimensional vector `Theta` contains the `d-1` angles of `X`, this function compute `X` in its rectangular coordinate representation.

Usage

```
Polar2Rec(R, Theta)
```

Arguments

`R`	The length of `X`.
`Theta`	A vector of length `d-1`, containing the angles of `X`.

Details

The formula corresponds to v=rho(theta) as in Lemma 1 of Tang and Li (2024). See also Anderson (2003). Note that when `d=2`, `V` will be `(sin(Theta),cos(Theta))`.

Value

A list of the following:

`X`	A vector in rectangular coordinate.
`V`	The directional vector of `X`. Note that `V` is always on the unit sphere.

References

Tang, Y. and Li, B. (2024), "A nonparametric test for elliptical distribution based on kernel embedding of probabilities," https://arxiv.org/abs/2306.10594 *Anderson, T. W. (2003). An Introduction to Multivariate Statistical Analysis. John Wiley & Sons, Inc. Huboken, New Jersey.*

Examples

```
R=2
Theta=c(pi/6,pi/3)
Polar2Rec(R,Theta)
```

PolarDerivative *Derivative of the polar coordinate transformation*

Description

This function compute the Jacobian matrix of the polar transformation theta=g(v), i.e., the transformation from the the rectangular coordinate representation of the directional vector to its angular representation.

Usage

```
PolarDerivative(v)
```

Arguments

`v`	A `d`-dimensional directional vector of length 1.

Details

See Lemma 3 of Tang and Li (2024).

Value

The Jacobian matrix of the polar transformation theta=g(v), with `d-1` rows and `d` columns.

References

Tang, Y. and Li, B. (2024), "A nonparametric test for elliptical distribution based on kernel embedding of probabilities," https://arxiv.org/abs/2306.10594

Examples

```
X=c(3,1,3)
V=X/sqrt(sum(X^2))
PolarDerivative(V)
```

`Rec2Polar` *Rectangular to polar coordinates*

Description

Given a `d`-dimensional vector `X` in rectangular coordinate, this function compute its polar coordinate `(R,Theta)`, where `R` is the length of `X` and the `(d-1)`-dimensional vector `Theta` contains the `d-1` angles of `X`.

Usage

```
Rec2Polar(X)
```

Arguments

`X`	A vector in rectangular coordinate. Suppose the dimension of `X` is `d`.

Details

The formula corresponds to theta=g(v) as in Lemma 1 of Tang and Li (2024). See also Anderson (2003). Note that when `d=2`, `V` will be `(sin(Theta),cos(Theta))`.

Value

A list of the following:

`R`	The length of `X`.
`Theta`	A vector of length `d-1`, containing the angles of `X`.

References

Tang, Y. and Li, B. (2024), "A nonparametric test for elliptical distribution based on kernel embedding of probabilities," https://arxiv.org/abs/2306.10594 *Anderson, T. W. (2003). An Introduction to Multivariate Statistical Analysis. John Wiley & Suns, Inc. Huboken, New Jersey.*

Examples

```
X=c(3,1,3)
Rec2Polar(X)
```

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