Package ‘contfrac’

October 12, 2022

Title Continued Fractions

Version 1.1-12

Author Robin K. S. Hankin

Description Various utilities for evaluating continued fractions.

Maintainer Robin K. S. Hankin <hankin.robin@gmail.com>

License GPL-2

URL https://github.com/RobinHankin/contfrac.git

NeedsCompilation yes

Repository CRAN

Date/Publication 2018-05-17 04:13:09 UTC

R topics documented:

---

as_cf *Approximates a real number in continued fraction form*

Description

Approximates a real number in continued fraction form using a standard simple algorithm

Usage

```
as_cf(x, n = 10)
```

Arguments

x	real number to be approximated in continued fraction form
n	Number of partial denominators to evaluate; see Notes

Note

Has difficulties with rational values as expected

Author(s)

Robin K. S. Hankin

See Also

CF,convergents

Examples

```
phi <- (sqrt(5)+1)/2
as_cf(phi,50)  # loses it after about 38 iterations ... not bad ...

as_cf(pi)  # looks about right
as_cf(exp(1),20)

f <- function(x){CF(as_cf(x,30),TRUE) - x}

x <- runif(40)
plot(sapply(x,f))
```

CF *Continued fraction convergents*

Description

Returns continued fraction convergent using the modified Lenz's algorithm; function `CF()` deals with continued fractions and `GCF()` deals with generalized continued fractions.

Usage

```
CF(a, finite = FALSE, tol=0)
GCF(a,b, b0=0, finite = FALSE, tol=0)
```

Arguments

`a,b`	In function `CF()`, the elements of `a` are the partial denominators; in `GCF()` the elements of `a` are the partial numerators and the elements of `b` the partial denominators
`finite`	Boolean, with default `FALSE` meaning to iterate Lenz's algorithm until convergence (a warning is given if the sequence has not converged); and `TRUE` meaning to evaluate the finite continued fraction
`b0`	In function `GCF()`, floor of the continued fraction
`tol`	tolerance, with default `0` silently replaced with `.Machine$double.eps`

Details

Function `CF()` treats the first element of its argument as the integer part of the convergent.

Function `CF()` is a wrapper for `GCF()`; it includes special dispensation for infinite values (in which case the value of the appropriate finite CF is returned).

The implementation is in C; the real and complex cases are treated separately in the interests of efficiency.

The algorithm terminates when the convergence criterion is achieved irrespective of the value of `finite`.

Author(s)

Robin K. S. Hankin

References

- W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling 1992. *Numerical recipes 3rd edition: the art of scientific computing*. Cambridge University Press; section 5.2 "Evaluation of continued fractions"
- W. J. Lenz 1976. Generating Bessel functions in Mie scattering calculations using continued fractions. *Applied Optics*, 15(3):668-671

See Also

`convergents`

Examples

```
phi <- (sqrt(5)+1)/2
phi_cf <- CF(rep(1,100))     # phi = [1;1,1,1,1,1,...]
phi - phi_cf     # should be small

# The tan function:
"tan_cf" <- function(z,n=20){
     GCF(c(z, rep(-z^2,n-1)), seq(from=1,by=2, len=n))
}
```

```
z <- 1+1i
tan(z) - tan_cf(z)   # should be small

# approximate real numbers with continued fraction:
as_cf(pi)

as_cf(exp(1),25)    # OK up to element 21 (which should be 14)

  # Some convergents of pi:
  jj <- convergents(c(3,7,15,1,292))
  jj$A / jj$B - pi


  # An identity of Euler's:
  jj <- GCF(a=seq(from=2,by=2,len=30), b=seq(from=3,by=2,len=30), b0=1)
  jj - 1/(exp(0.5)-1)   # should be small
```

`convergents` *Partial convergents of continued fractions*

Description

Partial convergents of continued fractions or generalized continued fractions

Usage

```
convergents(a)
gconvergents(a,b, b0 = 0)
```

Arguments

`a,b`	In function `convergents()`, the elements of `a` are the partial denominators (the first element of `a` is the integer part of the continued fraction). In `gconvergents()` the elements of `a` are the partial numerators and the elements of `b` the partial denominators
`b0`	The floor of the fraction

Details

Function `convergents()` returns partial convergents of the continued fraction

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{1}{a_3 + \cfrac{1}{a_4 + \cfrac{1}{a_5 + \ddots}}}}}$$

where `a` = $a_0, a_1, a_2, \ldots$ (note the off-by-one issue).

Function `gconvergents()` returns partial convergents of the continued fraction

$$b_0 + \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{b_4 + \cfrac{a_5}{b_5 + \ddots}}}}}$$

where `a` = $a_1, a_2, \ldots$

Value

Returns a list of two elements, `A` for the numerators and `B` for the denominators

Note

This classical algorithm generates very large partial numerators and denominators. To evaluate limits, use functions `CF()` or `GCF()`.

Author(s)

Robin K. S. Hankin

References

W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling 1992. *Numerical recipes 3rd edition: the art of scientific computing*. Cambridge University Press; section 5.2 "Evaluation of continued fractions"

See Also

CF

Examples

```
# Successive approximations to pi:

jj <- convergents(c(3,7,15,1,292))
jj$A/jj$B - pi     # should get smaller


convergents(rep(1,10))
```

Index